LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc., DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2013**

MT 5508/MT 5502 – LINEAR ALGEBRA

Date : 12/11/2013 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART A

ANSWER ALL THE QUESTIONS **(10 × 2 = 20 *marks*)**

1. If V is a vector space over a field F, show that $(-a)v = a(-v) = -(av)$ for $a \in F, v \in V$.
2. Show that the vectors $(1,1)$ and $(-3,2)$ in R^2 are linearly independent over R the field of real numbers.
3. Define a basis of a vector space.
4. Let $T: R^2 \to R^3$ be a vector space homomorphism defined by $T(a,b) = (a-b, b-a, -a)$ for all $a,b \in R$. Find nullity of T.
5. Normalize $(1+2i, 2-i, 1-i)$ in C^3 relative to the standard inner product.
6. Let $T \in A(v)$ and $\lambda \in F$. If λ is an eigenvalue of T, prove that $\lambda I - T$ is singular.
7. Define Nilpotent and Idempotent matrices.
8. Show that $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
9. Find the rank of the matrix A= $\begin{pmatrix} 1 & 5 & -7 \\ 2 & 3 & 1 \end{pmatrix}$ over the field of rational numbers.
10. Define unitary linear transformation.

PART B

ANSWER ANY FIVE QUESTIONS **(5 × 8 = 40 *marks*)**

11. Show that a nonempty subset W of a vector space V over F is a subspace of V if and only if $aw_1 + bw_2 \in W for all a, b \in F, w_1, w_2 \in W$
12. Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = (0)$.
13. If V is a vector space of dimension n, then prove that
    i) Any $n+1$ vectors in V are linearly dependent
    ii) Any set of n linearly independent vectors of V is basis of V.
14. If A and B are subspaces of a vector space V over F, prove that $(A+B)/B \cong A/A \cap B$.
15. State and prove Schwarz inequality.

16. If $\lambda \in F$ is an eigenvalue of $T \in A(V)$, then prove that for any polynomial $f(x) \in F[x]$, $f(\lambda)$ is an eigenvalue of $f(T)$.
17. Show that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.
18. If $T \in A(V)$ is skew-Hermitian, prove that all of its eigenvalues are pure imaginaries.

PART C

ANSWER ANY TWO QUESTIONS **(2 ⃞ 20 = 40 *marks*)**

19. a) If S and T are subsets of a vector space V over F,then prove that
    - i) S is a subspace of V if and only if $L(S) = S$.
    - ii) $S \subseteq T$ implies that $L(S) \subseteq L(T)$.
    - iii) $L(L(S)) = L(S)$.
    - iv) $L(S \cup T) = L(S) + L(T)$.

    b) If V is a vector space of finite dimension and W is a subspace of V, then prove that $\dim V/W = \dim V - \dim W$.

20. a) If V is a vector space of finite dimension ,and is the direct sum of its subspaces U and W, then prove that $\dim V = \dim U + \dim W$

    b) If U and V are vector spaces over F, and if T is a homomorphism of U onto V with kernel W, then prove that $U/W \cong V$.
21. State and prove Gram-Schmidt orthonormalization process.
22. a) Let $V = R^3$, and let $T \in A(V)$ be defined by $T(a_1, a_2, a_3) = (3a_1 + a_3, -2a_1 + a_2, -a_1 + 2a_2 + 4a_3)$. What is the matrix of T relative to the basis $v_1 = (1,0,1), v_2 = (-1,2,1), v_3 = (2,1,1)$?

    b) Investigate for what values of λ, μ the system of equations $x_1 + x_2 + x_3 = 6, x_1 + 2x_2 + 3x_3 = 10, x_1 + 2x_2 + \lambda x_3 = \mu$ over the rational field has a) no solution b) a unique solution c) an infinite number of solutions.

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